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Radiation processes in quantum systems with boundary

Yu M Sinyukov
State Standards Committee, Moscow, USSR

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Abstract. A study is made of the emission effects in quantum field systems contained in the regions of space with a non-stationary boundary. The back reaction of emission on the boundary is taken into account. The classical equations of motion of a system consisting of a free massless scalar field limited in space by a massive reflecting wall (mirror) are investigated in the two-dimensional case. The system is closed, i.e. external forces are absent. This system is quantised. The effect of the reaction of emission on the trajectory of a mirror is studied. The specific peculiarities of the stimulated emission processes in closed quantum systems with boundary are established and investigated. The results are related to hadron physics.

1. Introduction

Hawking's discovery (Hawking 1975) has stimulated quantum field-theoretical studies in spaces different from Minkowski space. A number of interesting physical effects associated primarily with the global properties of the manifolds considered have been established (see DeWitt 1975, Frolov and Serebriany 1979). In particular, the relationship between the mechanisms generating the Hawking emission in black holes and the particle production by an accelerating mirror has been demonstrated by Fulling and Davies (1976, 1977). The role of an external gravitational field in this case is played by the trajectory $z(t)$ of a mirror reflecting the quanta of a massless scalar field $\varphi(t, z)$ into the right-hand half-plane (figure 1). The analogue of Hawking's effect is attained with a certain class of trajectories with a null asymptote $\tau \equiv t+z =$ constant (Fulling and Davies 1977); it manifests itself in the thermal (Planck) character of the spectrum of scalar particles arising at the final stage (at J_R^+) as a result of the instability of an initial vacuum.

These effects for quantum field systems contained in flat non-stationary regions are also of interest for hadron physics. There are some hopes to solve the problem of confinement of quark–gluon matter within the framework of quantum chromodynamics (Callan *et al* 1979, Johnson 1979). Attempts have already been made to use the Hawking mechanism for extended hadron models (bags), to explain the thermodynamic behaviour of some hadron characteristics (Hosoya 1979). The effects concerned with vacuum instability may also serve as a basis for hydrodynamical models where the spectrum of final particles is defined by the character of the trajectory of the boundaries and expanding hadron matter (Gorenstein *et al* 1977, 1978).

There is, however, an important question that may not be disregarded without frustrating the attempts to use the effect that we consider in hadron physics. We mean taking into account the back reaction of the arising emission to a reflecting boundary.

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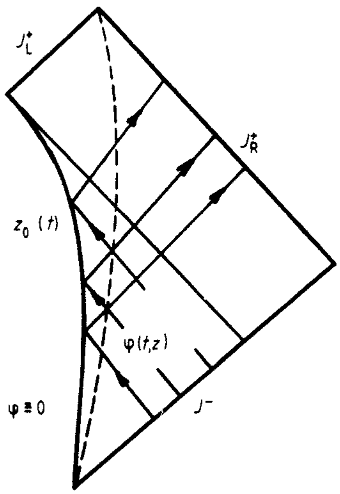

Figure 1. A Penrose diagram of the portion of two-dimensional Minkowski space to the right of a reflecting boundary. The mirror's trajectory $z_0(t)$ with a null asymptote $\tau=$ constant corresponds to the case when an external force is acting to accelerate the mirror (Fulling and Davies 1977). The light infinity of the past is denoted by J^-, and the right and left light infinities of the future by J_R^+ and J_L^+. The broken line indicates the trajectory of a mirror for a closed system consisting of a mirror and a field φ which is reflected by the mirror to the right (see § 2.5).

This problem is non-trivial, because the results quoted above are obtained by assuming that the character of the boundary trajectory is preassigned. This, in fact, means that the physical system is not closed and that there are no conserved quantities such as energy in the theory. Conversely, if we consider the system of a field plus a mirror as a closed one, we should be led to explicit expressions for conserved quantities, and should be able automatically to take account of the back reaction of the field to the boundary trajectory. It is the aim of this paper to state and solve this problem in the exactly solvable two-dimensional case. A field exists only on one side of the mirror. This case is interesting for physical applications, as it corresponds to the bag model ideology where the field on the outside of the bag is absent. We emphasise that in this paper an external force (corresponding to the gravitational field of the black hole) acting to accelerate the mirror is not considered.

In § 2, we formulate the model and find the classical equations of motion for a closed system. We also establish the laws of conservation of energy, momentum and angular momentum and give their explicit expressions in light-cone variables. In § 3, we study the Hamiltonian formulation of the model, Poisson brackets, and obtain classical solutions. In § 4, we quantise a closed relativistic system and introduce in- and out-states. To construct these states we diagonalise the energy operator of the closed system in two different ways. These ways correspond to different sets of creation and annihilation operators $a^{\pm}$, $b^{\pm}$, both quantisation methods being valid for all times. In the following we argue that in the limit $\tau\to\pm\infty$ the interaction between the mirror and the field tends to zero (e.g. the approximation of § 5 gives for the mirror acceleration $a\sim\exp(-t^2/c)$. Then the energy of the free field for $t\to-\infty$ is expressed in a diagonal form in terms of the operators $a^{\pm}$, and for $t\to+\infty$ in terms of the operators $b^{\pm}$. So the states induced by these operators are in- and out-states of the system.

Section 5 treats the S-matrix and particle production. For this reason we develop an approximate method by introducing the concept of a mirror trajectory for a closed quantum system taking into account the back reaction of emission. This approximation gives the connection between in- and out-operators. The nonlinearity of the problem is taken into account in this approximation as the dependence of Bogolubov transformation coefficients on the initial state of the system. We obtain the effect of anomalous (quasi-spontaneous) stimulated emission, which is important for astrophysics and hadron physics.

2. Formulation of the model and conservation laws

To describe in a systematic manner a closed system consisting of a field and a 'mirror wall' limiting at the left the half-space where the field is contained, it is necessary to regard the boundary coordinate as a dynamic variable. We consider a two-dimensional case: $x^\mu = (t, z)$. Let $\varphi(t, z)$ be a real, scalar massless field, m the mass of a 'wall', and $z_0(t)$ its coordinate. (The role of mass in the bag model where there is another boundary is placed by the surface tension σ (Hasenfratz and Kuti 1978).) The relativistic-invariant action for this system is

$$S = \int_{t_0}^{t_1} \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}z \left(\frac{1}{2} \frac{\partial \varphi}{\partial x^\mu} \frac{\partial \varphi}{\partial x_\mu} \right) - m \int_{t_0}^{t_1} \mathrm{d}t \, (1 - \dot{z}_0(t)^2)^{1/2}. \tag{1}$$

Variation of this action with respect to φ and z_0 leads to a Neumann equation at the boundary which has a derivative of the field. The mirror model is specified by a Dirichlet equation: $\varphi(t, z_0(t)) = \text{constant}$ (De Witt 1975, Fulling and Davies 1976). To obtain the equations of motion for this system we require that the last equation should be satisfied as an additional boundary condition. By varying the action (1) over φ, z_0 under this condition and putting $\delta S = 0$, we obtain:

$$\Box \varphi(t, z) = 0 \qquad \text{to the right of the boundary,} \tag{2}$$

$$\varphi(t, z_0(t)) = 0, \tag{3}$$

$$m \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\dot{z}_0}{(1 - \dot{z}_0^2)^{1/2}} \right) = \frac{1}{2} \frac{\partial \varphi}{\partial x_\mu} \frac{\partial \varphi}{\partial x^\mu} \bigg|_{z = z_0(t)}. \tag{4}$$

Equations (2)–(4) are a complete set of the equations of motion of a reflecting boundary and a field under a Dirichlet condition.

We now establish the conservation laws. The energy–momentum tensors for the field and mirror are

$$T^{\mu\nu}_{(\mathrm{f})} = \frac{\partial \varphi}{\partial x_\mu} \frac{\partial \varphi}{\partial x_\nu} - g^{\mu\nu} \frac{1}{2} \frac{\partial \varphi}{\partial x^\gamma} \frac{\partial \varphi}{\partial x_\gamma}, \tag{5}$$

$$T^{\mu\nu}_{(\mathrm{m})} = m \int \mathrm{d}\sigma \frac{\mathrm{d}x_0^\mu}{\mathrm{d}\sigma} \frac{\mathrm{d}x_0^\nu}{\mathrm{d}\sigma} \delta^{(2)}[x - x_0(\sigma)] \, \mathrm{d}\sigma = (g^{\mu\nu} \, \mathrm{d}x_0^\mu \, \mathrm{d}x_0^\nu)^{1/2} \tag{6}$$

where $g^{\mu\nu}$ in the coordinates $x^\mu = (t, z)$ has the form

$$g^{00} = -g^{11} = 1, \; g^{01} = g^{10} = 0$$

By integrating the relation $\partial_\mu T^{\mu\nu}_{(f)}=0$, resulting from (2), (5), over the two-dimensional (t, z) volume occupied by the field between times t_0 and $t_0+\mathrm{d}t$ and using equations (3), (4), we obtain the following relations:

$$\frac{\mathrm{d}E_{(f)}}{\mathrm{d}t}=-T^{0\nu}_{(f)}n_\nu=-\frac{\mathrm{d}}{\mathrm{d}t}\frac{m}{(1-\dot{z}_0^2)^{1/2}} \qquad \frac{\mathrm{d}}{\mathrm{d}t}(E_{(f)}+E_{(m)})=0 \tag{7}$$

$$\frac{\mathrm{d}P_{(f)}}{\mathrm{d}t}=-T^{1\nu}_{(f)}n_\nu=-\frac{\mathrm{d}}{\mathrm{d}t}\frac{m\dot{z}_0}{(1-\dot{z}_0^2)^{1/2}}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}(P_{(f)}+P_{(m)})=0. \tag{8}$$

Here $n_\mu=(\dot{z}_0,-1)$ is the vector proportional to the normal for the boundary trajectory; $E_{(f)}$, $P_{(f)}$ and $E_{(m)}$, $P_{(m)}$ are the energy and the momentum of the field and of the mirror, respectively.

In a similar way we can show the conservation of the total angular momentum $M^{\mu\nu}_{(f)}+M^{\mu\nu}_{(m)}$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{-\infty}^{z_0(t)}(x^\nu T^{\mu 0}_{(f)}-x^\mu T^{\nu 0}_{(f)})\,\mathrm{d}z+x_0^\nu P^\mu_{(m)}-x_0^\mu P^\nu_{(m)}\right)=0. \tag{9}$$

It is convenient to investigate the equations of motion and to quantise the system in terms of light-cone variables

$$x^+\equiv\tau=(1/\sqrt{2})(t+z), \qquad x^-\equiv x=(1/\sqrt{2})(t-z), \qquad g^{+-}=g^{-+}=1,$$

$$g^{++}=g^{--}=0. \tag{10}$$

Equation (2) in these variables has the solution

$$\varphi=f(\tau)+g(x) \tag{11}$$

and equations (3), (4) are much simplified,

$$f(\tau)=-g[x_0(\tau)], \tag{12}$$

$$\dot{f}(\tau)g'[x_0(\tau)]=\frac{m}{\sqrt{2}}\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{1}{(\dot{x}_0(\tau))^{1/2}}, \tag{13}$$

where $\dot{x}_0(\tau)=(1-\dot{z}_0(t))(1+\dot{z}_0(t))^{-1}$, $0<\dot{x}_0(\tau)<\infty$. The derivatives with respect to time and space variables are denoted by dots and primes.

The integrals of motion $\mathscr{P}$

$$\mathscr{P}(\xi)=\int_\Sigma \mathrm{d}\Sigma_\nu(T^{\mu\nu}_{(f)}+T^{\mu\nu}_{(m)})\xi_\mu \tag{14}$$

with Killing vectors ξ in (τ, x)-space, $\xi^{(M)}_\mu=(0,1)$, $\xi^{(P)}_\mu=(1,0)$, $\xi^{(M)-+}_\mu=\delta^-_\mu x^+-\delta^+_\mu x^-$, have the form

$$P^-\equiv H=\frac{m}{\sqrt{2}}(\dot{x}_0(\tau))^{1/2}+\int_\tau^\infty(\dot{f}(\tau'))^2\,\mathrm{d}\tau', \tag{15}$$

$$P^+\equiv P=\frac{m}{\sqrt{2}}\frac{1}{(\dot{x}_0(\tau))^{1/2}}+\int_{-\infty}^{x_0(\tau)}(g'(x))^2\,\mathrm{d}x, \tag{16}$$

$$M^{-+}\equiv M=\frac{m}{\sqrt{2}}\left(\sqrt{\dot{x}}\tau-\frac{x_0}{\sqrt{\dot{x}_0}}\right)+\int_\tau^\infty\mathrm{d}\tau'\,\tau'(\dot{f}(\tau))^2-\int_{-\infty}^{x_0(\tau)}\mathrm{d}x\,x(g'(x))^2. \tag{17}$$

The integration in (14) is over the surface Σ shown in figure 2. Using the relations (12), (13), we can see immediately that the quantities (15)–(17) are conserved.

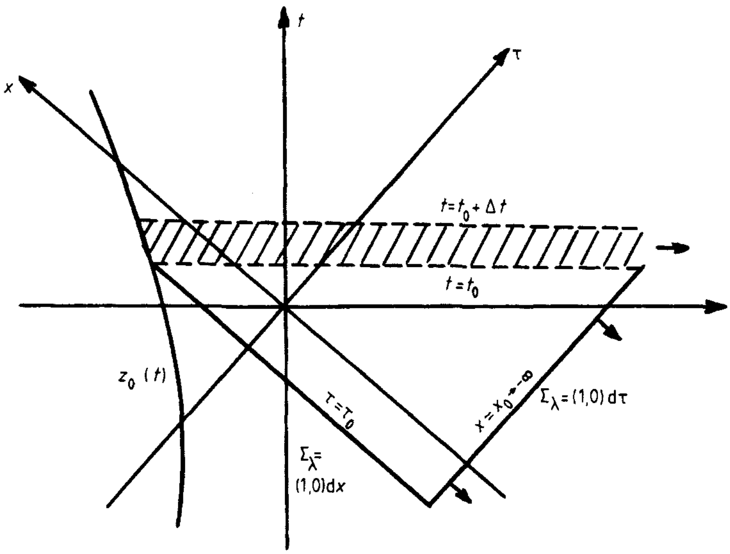

Figure 2. Cauchy surface for the invariants of motion. The broken lines indicate space-like surfaces $t=$ constant ($z_0(t)<z<\infty$); the full line corresponds to a light-like surface Σ: $\tau=\tau_0$ ($-\infty<x<x_0(\tau)$), $x=x^*\to-\infty$ ($\tau_0<\tau<\infty$).

3. Hamiltonian formalism and Poisson brackets

The nonlinear set of equations (12), (13) defines (to an accuracy of constants) the function of the field φ, and the trajectory $x_0(\tau)$ by assigning the function $f(\tau)$:

$$\varphi=f(\tau)-f[\tau_0(x)], \qquad \tau_0(x) \text{ is the function inverse of } x_0(\tau), \tag{18}$$

$$x_0(\tau)=\frac{2}{m^2}\int_{-\infty}^{\tau}\mathrm{d}\tau'\left(\int_{-\infty}^{\tau'}\mathrm{d}\tau''(\dot{f}(\tau''))^2\right)^2+\frac{2}{P}\int_{-\infty}^{\tau}\mathrm{d}\tau'\int_{-\infty}^{\tau'}\mathrm{d}\tau''(\dot{f}(\tau''))^2-Q(\tau), \tag{19}$$

where

$$P=(m/\sqrt{2})(\dot{x}_0(-\infty))^{-1/2}, \tag{20}$$

$$Q(\tau)=-(m^2/2P^2)\tau+Q_0, \qquad Q_0=\text{constant}. \tag{21}$$

It is easy to see that in the classical case equation (13) responsible for the back reaction of a field to a mirror involves no null asymptote: $\tau_0(x)\to$ constant, $x\to\infty$. According to (13), (15), this would correspond to an infinite energy of the system.

For quantisation purposes we now formulate the classical equations in Hamiltonian form and find the Poisson brackets that we need. For this reason we change to new variables in which the Hamiltonian H is diagonal. We put

$$f(\tau)=\frac{1}{(4\pi)^{1/2}}\int_0^{\infty}\frac{a^+(\kappa)}{\sqrt{\kappa}}\mathrm{e}^{\mathrm{i}\kappa\tau}\,\mathrm{d}\kappa+\frac{1}{(y\pi)^{1/2}}\int_0^{\infty}\frac{a^-(\kappa)}{\sqrt{\kappa}}\mathrm{e}^{-\mathrm{i}\kappa\tau}\,\mathrm{d}\kappa \tag{22}$$

where $(a^+(\kappa))^*=a^-(\kappa)$ because the field is real. We use (22) and the equation of motion to express H, P, M in terms of $a(\kappa)$. Then

$$H=\tfrac{1}{2}\int_0^\infty \mathrm{d}\kappa\,\kappa(a^+(\kappa)a^-(\kappa)+a^-(\kappa)a^+(\kappa))+\frac{m^2}{2p}, \tag{23}$$

$$P=p, \tag{24}$$

$$M=(m^2/2p)\tau+\tfrac{1}{2}(PQ+Qp)+\frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau\,\tau\int\int_0^\infty \mathrm{d}\kappa_1\,\mathrm{d}\kappa_2(\kappa_1\kappa_2)^{1/2}$$
$$\times\{a^+(\kappa_1)a^-(\kappa_2)\exp[\mathrm{i}(\kappa_1-\kappa_2)\tau]+a^-(\kappa_1)a^+(\kappa_2)\exp[-\mathrm{i}(\kappa_1-\kappa_2)\tau]\}. \tag{25}$$

As independent real coordinates $q(\kappa,\tau)$ and momenta $p(\kappa,\tau)$ we choose

$$q(\kappa,\tau)=(\mathrm{i}/2\sqrt{\kappa})(a^-(\kappa)\,\mathrm{e}^{-\mathrm{i}\kappa\tau}-a^+(\kappa)\,\mathrm{e}^{\mathrm{i}\kappa\tau}),\qquad p(\kappa,\tau)=\sqrt{\kappa}(a^-(\kappa)\,\mathrm{e}^{-\mathrm{i}\kappa\tau}+a^+(\kappa)\,\mathrm{e}^{\mathrm{i}\kappa\tau}). \tag{26}$$

The Hamiltonian (23) in these variables has the form

$$H=\tfrac{1}{4}\int_0^\infty (p^2(\kappa,\tau)+4\kappa^2q^2(\kappa\tau))\,\mathrm{d}\kappa+\frac{m^2}{2P}. \tag{27}$$

The Hamiltonian equations and the Poisson brackets for variables $q(\kappa,\tau)$ and $p(\kappa,\tau)$ are of the usual form

$$\dot{q}(\kappa,\tau)=\{q(\kappa,\tau),H\}=\frac{\delta H}{\delta p(\kappa,\tau)},\qquad \{q(\kappa_1,\tau)p(\kappa_2,\tau)\}=\delta(\kappa_1-\kappa_2),$$
$$\dot{p}(\kappa,\tau)=\{p(\kappa,\tau),H\}=-\frac{\delta H}{\delta\bar{q}(\kappa,\tau)},\qquad \{q(\kappa_1,\tau),q(\kappa_2,\tau)\}=\{p(\kappa_1,\tau),p(\kappa_2,\tau)\}=0. \tag{28}$$

It is easy to see that the cyclic variable Q (21) is the canonical conjugate of P:

$$\dot{Q}(\tau)=\partial H/\partial P=\{Q(\tau),H\}. \tag{29}$$

We have the Poisson brackets for variables $Q(\tau)$, P

$$\{Q,P\}=1,\qquad \{P,q(\kappa,\tau)\}=\{P,p(\kappa,\tau)\}=0,\qquad \{Q,q(\kappa,\tau)\}=\{Q,p(\kappa,\tau)\}=0. \tag{30}$$

Using the form of the function $f(\tau)$ in the new variables,

$$f(\tau)=1/(4\pi)^{1/2}\int_0^\infty \frac{p(\kappa,\tau)}{\kappa}\,\mathrm{d}\kappa, \tag{31}$$

we can give the solution (19) for $x_0(\tau)$ in terms of $p(\kappa,\tau)$, $g(\kappa,\tau)$ and see that

$$\frac{\mathrm{d}}{\mathrm{d}\tau}x_0(q_\kappa,p_\kappa,P,Q,\tau)=\frac{\partial x_0}{\partial\tau}+\{x_0,H\}. \tag{32}$$

Direct calculation using the relations (27), (28), (30) leads us to correct Poincaré relations for the Poisson brackets of (23)–(25):

$$\{M,H\}=-H,\qquad \{M,P\}=P,\qquad \{H,P\}=0. \tag{33}$$

We conclude this section by indicating an alternative method also resulting in a diagonalisation of the total energy $E=(1/(\sqrt{2})(P^{+}+P^{-})$ of the system.

Just as we have expressed the solutions (18), (19) of equations (12), (13) by assigning the function $f(\tau)$, we can obtain solutions to these equations by expressing them in terms of the functions $g(\alpha)$. Then

$$\varphi=g(x)-g(x_0(\tau)), \tag{34}$$

$$-\tau_0(x)=\frac{2}{m^2}\int_x^{\infty}\mathrm{d}x'\left(\int_{x'}^{\infty}\mathrm{d}x''(g'(x''))^2\right)^2+\frac{1}{h}\int_x^{\infty}\mathrm{d}x'\int_{x'}^{\infty}\mathrm{d}x''(g'(x''))^2+S(x), \tag{35}$$

where

$$h=(m/(\sqrt{2})(\dot{x}_0(+\infty))^{1/2}\qquad(0<h<\infty) \tag{36}$$

$$S(x)=-m^2x/2h^2+S_0,\qquad S_0=\text{constant}. \tag{37}$$

By representing the function $g(x)$ as

$$g(x)=\frac{1}{(4\pi)^{1/2}}\int_0^{\infty}\frac{b^{+}(q)}{\sqrt{q}}\mathrm{e}^{\mathrm{i}qx}\,\mathrm{d}q+\frac{1}{(4\pi)^{1/2}}\int_0^{\infty}\frac{b^{-}(q)}{\sqrt{q}}\mathrm{e}^{-\mathrm{i}qx}\,\mathrm{d}q \tag{38}$$

under the condition that the field is real $(b^{+}(q))^{*}=b^{-}(q)$, and proceeding along the lines indicated above, we arrive at

$$P^{+}=\int_0^{\infty}\mathrm{d}q\,qb^{+}(q)b^{-}(q)+\frac{m^2}{2h}, \tag{39}$$

$$P^{-}=h, \tag{40}$$

$$-M=\int_{-\infty}^{\infty}\mathrm{d}x\,x(g'(x))^2+\frac{m^2}{2h}x+\tfrac{1}{2}(hS+Sh). \tag{41}$$

P^{+} is used here as the Hamiltonian of the system.

By changing next to independent variables, expressed in terms of $b(q)$, such as coordinates and momenta, we can easily obtain commutation relations such as (28), (30), where the conjugate pair of variables Q, P correspond to S, h, and establish that the relations (33) are valid.

4. Quantisation of the system

The quantisation is done via the correspondence principle, the Poisson brackets being replaced by commutators:

$$\mathrm{i}\{A,B\}\to[A,B].$$

This procedure results in the following commutation relations:

$$\begin{aligned}&[a^{-}(\kappa),a^{+}(\kappa_2)]=\delta(\kappa_1-\kappa_2),\qquad [a^{+},a^{+}]=[a^{-},a^{-}]=0,\\&[Q,P]=\mathrm{i},\qquad [Q,a^{\pm}]=[P,a^{\pm}]=0.\end{aligned} \tag{42}$$

In a similar way we obtain for the variables introduced through (36)–(38)

$$\begin{aligned}&[b^{-}(q_1)b^{+}(q_2)]=\delta(q_1-q_2),\qquad [b^{+},b^{+}]=[b^{-},b^{-}]=0\\&[S,h]=\mathrm{i},\qquad [S,b^{\pm}]=[h,b^{\pm}]=0.\end{aligned} \tag{43}$$

Relations (33) or direct calculations of (15)–(17) with (42) taken into account lead us to a correct Poincaré algebra for quantum theory

$$[M, H] = -\mathrm{i}H, \qquad [M, P] = \mathrm{i}P, \qquad [H, P] = 0. \tag{44}$$

We now determine the vacuum state of the system

$$a^-(\kappa)|\phi_0\rangle = 0 \tag{45}$$

and regard the observables H, P, M, $x_0(\tau)$ as normally ordered. In particular,

$$H = \int_0^\infty \mathrm{d}\kappa\, \kappa a^+(\kappa)a^-(\kappa) + \frac{m^2}{2P}. \tag{46}$$

The invariant mass of the system, M^2, is

$$M^2 = 2P\int_0^\infty \mathrm{d}\kappa\, \kappa a^+(\kappa)a^-(\kappa) + m^2. \tag{47}$$

The vacuum state has a non-zero momentum $P = P^+$. This is so because the system possesses a non-zero invariant mass even in the absence of excitations in the system, as follows from (47). This observation can be explained by the fact that a mirror of mass m is, by the very statement of the problem, an object with a fixed number of degrees of freedom. As a result, the quantisation yields two separate conjugate variables Q and P satisfying the quantum mechanical commutation relation. The variable P has the significance of a total momentum of the system, P^+, in light-cone variables, and Q is the 'coordinate' connected linearly with the coordinate characterising the centre-of-mass system. The space of states of a quantum system may therefore be represented as a superposition of states

$$|\phi\rangle = |\phi_a\rangle|\phi_p\rangle \tag{48}$$

where $|\phi_a\rangle$ are the vectors of Fock space constructed in a standard manner by the action of the operators $a^+(\kappa)$ on the vacuum, and the vectors $|\phi_p\rangle$ correspond to the free motion of the system as a whole.

It is easy to see that the sum of two terms in the Hamiltonian (46) corresponds to the sum of energy contributions from a free massless field φ and a mirror of mass m at $t \to -\infty$:

$$P^-_{(\mathrm{f})}(-\infty) = \sqrt{2}E_{(\mathrm{f})}(-\infty) = \int_0^\infty \mathrm{d}\kappa\, \kappa a^+(\kappa)a^-(\kappa),$$
$$P^-_{(\mathrm{m})}(-\infty) = m^2/2P.$$

In other words, the Hamiltonian of the system was diagonalised by going over to the in-operators $A^\pm_{\mathrm{in}}(\kappa)$ of the field $\varphi(\tau, x)$. The corresponding field φ_{in} is a free massless scalar field limited at the left by a mirror barrier moving with constant velocity $(\dot{x}_0(-\infty))$. The first term in (46) corresponds to the translation generator of the field φ at the light infinity of the past:

$$H_{(\mathrm{f})}(J^-) = P^-_{(\mathrm{f})}(A_{\mathrm{in}}), \qquad A^\pm_{\mathrm{in}}(\kappa) = a^\pm(\kappa). \tag{50}$$

Similarly, the operators $b^\pm(q)$, diagonalising the total energy of the system and introduced through (38), correspond to out-operators, so that

$$H_{(\mathrm{f})}(J^+) = P^+_{(\mathrm{f})}(A_{\mathrm{out}}), \qquad A^\pm_{\mathrm{out}}(q) = b^\pm(q), \tag{51}$$

where $P^+_{(\mathrm{f})}$ corresponds to the first term in (39).

We conclude by mentioning that the states of the closed system (48) introduced by us are in-states. Final out-states are constructed in a similar way from the superposition of states such as

$$|\phi_{\text{out}}\rangle = |\phi_b\rangle|\phi_h\rangle. \tag{52}$$

5. *S*-matrix, particle production and the back reaction of emission

To establish the connection between the operators A_{in} and A_{out} we make use of the relations (18), (34):

$$\varphi(\tau, x) = f_a(\tau) - f_a[\tau_0(x)] = g_b(x) - g_b[x_0(\tau)] \tag{53}$$

where the functions f_a are expressed in terms of the variables $a^{\pm}(\kappa)$ through the use of (19), (22), and the g_b are expressed in terms of the variables $b^{\pm}(q)$ according to (35), (38).

We thus have

$$A^{+}_{\text{out}}(q) = -\frac{\sqrt{q}}{2\pi}\left(\int_0^{\infty} \mathrm{d}\kappa \int_{\infty}^{\infty} \mathrm{d}\tau \frac{A^{+}_{\text{in}}(\kappa)}{\sqrt{\kappa}} \exp\left[\mathrm{i}\kappa\tau - \mathrm{i}q\hat{x}_0(\tau)\right]\hat{\dot{x}}\right.$$
$$\left. - \int_0^{\infty} \mathrm{d}\kappa \int_{-\infty}^{\infty} \mathrm{d}\tau \frac{A^{+}_{\text{in}}(\kappa)}{\sqrt{\kappa}} \exp\left[-\mathrm{i}\kappa\tau + \mathrm{i}q\hat{x}_0(\tau)\right]\hat{\dot{x}}_0\right),$$

$$A^{-}_{\text{out}}(q) = -\frac{\sqrt{q}}{2\pi}\left(\int_0^{\infty} \mathrm{d}\kappa \int_{-\infty}^{\infty} \mathrm{d}\tau\, \hat{\dot{x}} \exp\left(-\mathrm{i}qx - \mathrm{i}\kappa\tau\right) \frac{A^{-}_{\text{in}}(\kappa)}{\sqrt{\kappa}}\right.$$
$$\left. - \int_0^{\infty} \mathrm{d}\kappa \int_{-\infty}^{\infty} \mathrm{d}\tau\, \hat{\dot{x}} \exp\left[\mathrm{i}q\hat{x}_0(\tau) + \mathrm{i}\kappa\tau\right] \frac{A^{+}_{\text{in}}(\kappa)}{\sqrt{\kappa}}\right).$$

The expressions for $\hat{x}_0(\tau)$ and $\hat{\dot{x}}_0(\tau)$ in terms of $A^{\pm}_{\text{in}}(\kappa) = a^{\pm}(\kappa)$ are obtained from (19), (22) and considered to be normally ordered.

The investigation of a complete *S*-matrix induced by the relations (50) is a difficult enough problem, because the relationship between in and out-operators is essentially nonlinear. In the present paper we propose an approximate method of solving this question by introducing a mirror trajectory taking into account the reaction of emission.

The mirror trajectory operator $\hat{x}_0(\tau)$ in our model is dependent upon the field operators A_{in}, and this indicates that there is interaction in the system at finite times. Suppose that a closed system is in some state $|\phi\rangle$. The average in the vector $|\phi\rangle$ of the operator $\hat{x}(\tau)$ is then an analogue of the classical trajectory:

$$\bar{x}_{\phi}(\tau) = \langle\phi|\hat{x}(\tau)|\phi\rangle. \tag{55}$$

We now consider the limit when the mirror mass tends to infinity. Since $M^2 \to \infty$ in this case, the motion of the system as a whole may be described classically, and the relevant operators Q and P may be considered to be commuting ones. Next, it is easy to see from (19) and (22) that

$$\frac{\langle\phi|(\hat{x}(\tau) - \bar{x}_{\phi}(\tau))^2|\phi\rangle}{(x_{\phi}(\tau))^2} \sim \frac{\Lambda^2}{m^2}, \qquad m \to \infty, \Lambda = \text{constant},$$

so that we can disregard the dispersion for the trajectory $\bar{x}_{\phi}(\tau)$ within this limit.

Without going into detailed estimations of the expansions of physical quantities in powers of $1/m$, we shall proceed from intuitively clear considerations. We can then think that at sufficiently large m (as compared with the field energy), the interaction of a massive boundary with a field when a closed system is in a state $|\phi\rangle$ is approximately reduced to the presence of a classical trajectory $\bar{x}_\phi(\tau)$ limiting the space occupied by a quantised field. As a matter of fact, $x_\phi(\tau)$ in the approximation under consideration is an effective trajectory, taking account of the reaction of a quantised field which is in an initial state $|\phi\rangle$, to a mirror which has an initial velocity $\dot{x}(-\infty)$.

This approximation allows one to linearise the relations (54) connecting in and out-operators. Suppose that the field is an initial state $|\phi_{\text{in}}\rangle$:

$$|\phi_{\text{in}}\rangle = \frac{1}{\sqrt{n!}}\int_0^\infty \chi_n(\kappa_1 \ldots \kappa_n) \prod_{i=1}^{n} a^+(\kappa_i)\, d\kappa_i |\phi_0, \text{in}\rangle$$

where $\chi_n(\ldots \kappa_i \ldots)$ are the normalised wavefunctions of a system of n identical particles. Let $|\phi_{\text{out}}\rangle$ be some final state of a field:

$$|\phi_{\text{out}}\rangle = \frac{1}{\sqrt{m!}}\int_0^\infty \chi'_m(q_1 \ldots q_m) \prod_{j=1}^{m} b^+(q_j)\, dq_j |\phi_0, \text{out}\rangle.$$

We can then use (53) to obtain the following expressions for the S-matrix elements:

$$S_{\chi'_m,\chi_n} = \int_0^\infty\!\!\int \langle\phi_0, \text{out}|b_\phi^-(q_1)\ldots b_\phi^-(q_m)a^+(\kappa_1)\ldots a^+(\kappa_n)|\phi_0, \text{in}\rangle \frac{(\chi'_m)^*\chi_n}{(n!m!)^{1/2}} \prod_{i,j=1}^{n,m} d\kappa_i\, dq_j \tag{56}$$

where

$$b_\phi^-(q) = \int_0^\infty \alpha_\phi(q,\kappa)a^-(\kappa)\, d\kappa - \int_0^\infty \beta_\phi(q,\kappa)a^+(\kappa)\, d\kappa \tag{57}$$

and the Bogolubov transformation coefficients

$$\left.\begin{matrix}\alpha_\phi(q,\kappa)\\ \beta_\phi(q,\kappa)\end{matrix}\right\} = \mp\frac{1}{2\pi}\left(\frac{q}{\kappa}\right)^{1/2}\int_{-\infty}^\infty \exp[\mp i\kappa\tau + iq\bar{x}_\phi(\tau)]\dot{\bar{x}}_\phi(\tau)\, d\tau \tag{58}$$

with the index ϕ corresponding to the state $|\phi\rangle = |\phi_{\text{in}}\rangle$.

Direct calculation can show that the transformation (57), (58) is a canonical one irrespective of the choice of the initial state $|\phi\rangle$.

Using (19), (22), (57), (58), we find that if the system (field) at J^- has a vacuum as its state $|\phi\rangle = |\phi_0, \text{in}\rangle$, spontaneous emission of radiation at J^+ is zero:

$$\langle\phi_0, \text{in}|\hat{N}_q^{\text{out}}|\phi_0, \text{in}\rangle = \langle\phi_0, \text{in}|b_{\phi_0}^+(q)b_{\phi_0}^-(q)|\varphi_0, \text{in}\rangle = \int_0^\infty |\beta_{q\kappa}|^2\, d\kappa = 0. \tag{59}$$

The vacuum of a field is thus stable, and $|\phi_0, \text{out}\rangle = |\phi_0, \text{in}\rangle \equiv |0\rangle$ in the expression (56) for S-matrix elements.

The non-trivial result produced by the model is the presence of an anomalous essentially quantum contribution to stimulated emission if the initial state of the system is different from the vacuum. Suppose

$$|\phi\rangle = (N_0!)^{-1/2}\int_0^\infty \prod_{i=1}^{N_0} \chi_{P_0}(\kappa_i)a^+(\kappa_i)\, d\kappa_i|0\rangle \tag{60}$$

where $\chi_{p_0}(\kappa_i)$ are the normalised wavepackets with an average value of the energy p_0. Then $\bar{x}_\phi(\tau)$ is (in first order of $N_0 p_0/m$)

$$\bar{x}_\phi(\tau) = \dot{x}_0(-\infty)\tau Q_0 + \lim_{\lambda\to\infty} N_0 \frac{(2\dot{x}_0(-\infty))^{1/2}}{\pi m} \int\int_0 \mathrm{d}\kappa_1\,\mathrm{d}\kappa_2 \frac{(\kappa_1\kappa_2)^{1/2}}{(\kappa_2-\kappa_1)^2}$$

$$\times \chi^*_{p_0}(\kappa_1)\chi_{p_0}(\kappa_2)\{\exp[\mathrm{i}(\kappa_2-\kappa_1)^2][1-\mathrm{i}(\kappa_2-\kappa_1)(\tau+\lambda)] - \exp[-\mathrm{i}(\kappa_2-\kappa_1)\tau]\} \quad (61)$$

where $N_0 = N_{J^-}$ is the initial number of particles at J^-. The third term in (61) corresponds to the back reaction of the field to the mirror.

The density of the number of particles with energy q, emitted to J^-, is

$$\langle\phi|\hat{N}_q^{\mathrm{out}}|\phi\rangle = \int_0^\infty |\beta_\phi(q,\kappa)|^2\,\mathrm{d}\kappa,$$

$$+ N_0 \int\int_0^\infty \mathrm{d}\kappa_1\,\mathrm{d}\kappa_2\,\chi^*(\kappa_1)\chi(\kappa_2)(\alpha^*_\phi(q,\kappa_1)\alpha_\phi(q,\kappa_2) + \beta^*_\phi(q,\kappa_1)\beta_\phi(q,\kappa_2)). \quad (62)$$

The total number of particles emitted to J^- is

$$N_{J^+} = N_{J^-} + \int\int_0^\infty |\beta_\phi(q,\kappa)|^2\,\mathrm{d}\kappa\,\mathrm{d}q$$

$$+ 2N_{J^-}\int\int\int_0^\infty \mathrm{d}\kappa_1\,\mathrm{d}\kappa_2\,\mathrm{d}q\,\chi^*(\kappa_1)\chi(\kappa_2)\beta^*_\phi(q,\kappa_1)\beta_\phi(q,\kappa_2). \quad (63)$$

To go over to (63), we used the relation following from (58)

$$\int_0^\infty (\alpha^*_\phi(q,\kappa_1)\alpha_\phi(q,\kappa_2) - \beta^*_\phi(q,\kappa_1)\beta_\phi(q,\kappa_2))\,\mathrm{d}q = \delta(k_2-\kappa_1).$$

All the terms in (63) are positive. Thus $N_{J^+} > N_{J^-}$, and unless the initial state is a vacuum, the mirror will always produce particles. The third term in (63), proportional to the initial number of particles, is characteristic for stimulated emission and has, in fact, a classical nature (Fulling and Davies 1977). The status of the second term on the right-hand side of (63) is not ordinary. It vanishes at $N_0 = 0$, as does the third term, because $\beta_\phi(q,\kappa)$ is dependent upon N_0 (according to (61), the mirror in this case travels with a constant velocity $\dot{x}_0(-\infty)$). This implies that the second term is also responsible for stimulated emission. Its structure, however, exactly corresponds to the quantities defining the spontaneous emission of particles by a mirror which moves along a given trajectory $x(\tau) = \bar{x}_\phi(\tau)$ and makes a major contribution to particle flux at J^+ (Fulling and Davies 1977). This anomalous, quasi-spontaneous character of stimulated emission is one of the most interesting peculiarities of the behaviour of closed quantum systems with a boundary.

The total reaction of initial and stimulated emission at the boundary is described by the third term in the expression (61) for an effective trajectory. As mentioned above, an analogue of the Hawking effect in the 'mirror model' is attained with a certain class of trajectories having a null asymptote. Analysis of (61), (19), however, shows that in a closed quantum system the trajectory $\bar{x}_\phi(\tau)$, taking into account the reaction of emission, occurs for all τ:

$$\bar{x}_\phi(\tau) \to \begin{matrix} v_1\tau, & \tau \to +\infty, \\ v_0\tau, & \tau \to -\infty, \end{matrix} \qquad \infty > \dot{\bar{x}}_\phi(\tau) > 0. \tag{64}$$

Consequently, the null asymptote (analogue of the event horizon) does not occur here. The question that arises immediately, of whether taking account of the back reaction of emission in the analogue to the Hawking effect is analogous to taking account of the emission on the metric in the Hawking effect, requires separate treatment.

We now find the total energy $\mathscr{E}$ of an anomalous quantum stimulated emission. The Bogolubov transformation coefficients are supposed to be distributions defined on finite functions $F(q, \kappa)\theta(\kappa)\theta(q)$ (including $F(q, \kappa) = \text{constant}$). It is not difficult to show that for the trajectories of the type (64) the Bogolubov transformation coefficients $\beta_\phi(q, \kappa)$ are regular in q, κ anywhere, with the exception of the point $q = \kappa = 0$. Direct calculation of $\mathscr{E}$, using the standard rules of manipulation with distributions and taking into consideration the unambiguity of the function $\tau_\phi(x) = x_\phi^{-1}(\tau)$ traced from (64), yields

$$\mathscr{E} = \int_0^\infty \mathrm{d}\kappa\, \mathrm{d}q\, q\, |\beta_\phi(q, \kappa)|^2 = \frac{1}{24\pi} \int_{-\infty}^\infty \mathrm{d}\tau \frac{\dddot{x}_\phi(\tau)}{(\dot{x}_\phi(\tau))^2} = \frac{1}{12\pi} \int_{-\infty}^\infty \frac{(\ddot{x}_\phi)^2}{(\dot{x}_\phi)^3}\, \mathrm{d}\tau. \tag{65}$$

For trajectories of the class (64) the result (65) is formally the same as the expression for the energy of spontaneous emission at a given trajectory of a mirror, which is obtained by the covariant point splitting technique in Fulling and Davies (1976).

Suppose that the initial state of the system is described by the vector $|\phi\rangle$. Now, if as $\chi_{p_0}(\kappa)$ we choose wavepackets of the Gaussian form

$$\chi_{p_0}(\kappa) \sim \exp[-(\kappa - p_0)^2/D^2],$$

the trajectory (61) has the form

$$\bar{x}_\phi(\tau) = v_0\tau + \frac{N_0(2v_0)^{1/2}}{\pi m} p_0 \left(\pi\tau + \frac{(2\pi)^{1/2}}{D}\, {}_1F_1(-\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}\tau^2 D^2)\right) - Q_0 \tag{66}$$

where ${}_1F_1(a, b, z)$ is the Kummer function. Using (65), we obtain finally

$$\mathscr{E} = \frac{4D}{3\pi^{3/2}\sqrt{v_0}} \left(\frac{E^0_{(\mathrm{f})}}{m}\right)^2 \left(\sqrt{v_0} + 2\sqrt{2}\frac{E^0_{(\mathrm{f})}}{m}\right)^{-3} \tag{67}$$

where

$$E^0_{(\mathrm{f})} = N_0 \left(p_0 + \frac{D \exp(-2p_0^2/D^2)}{(2\pi)(1 - \mathrm{erf}(-p_0/D))}\right)$$

is the initial energy of the field. ($E^0_{(\mathrm{m})}$ is the initial energy of the mirror.)

The result (67) lends itself to a clear physical interpretation. It implies that a considerable contribution comes from the quantum stimulated emission only at $v_0 \ll 1$, i.e. when the initial velocity of a mirror is directed towards the field and is high: $\dot{z}_0 \to 1$. This is accompanied by the deceleration of the mirror by the field. The mirror's energy lost in the process is spent in exciting the field oscillators, resulting in an emission of new quanta. According to (67), the highest possible emission under this process is $\mathscr{E} \sim E^0_{(\mathrm{m})}$.

6. Conclusions

We have considered the specific peculiarities of the radiation processes in closed quantum systems with boundary. The discussion refers to the exactly solvable two-dimensional scalar field case. We think however that the above peculiarities of the radiation processes in the systems of interest are of a general nature. The physical reason for this (quasi-spontaneous) emission is an accelerated motion of the induced surface bound charges (currents) guaranteeing the boundary condition (type $\varphi = 0$). In closed systems the acceleration is due to the pressure of the field on the boundary surface.

Closed systems with a boundary are apparently realised in QCD: the bags (Callan *et al* 1979, Johnson 1979). The boundary conditions in the bags correspond to a screening by induced colour magnetic poles and currents; these sources are present in QCD. The results of this paper indicate another mechanism of quark and gluon production in non-stationary bags.

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